Nonlinear theory of fractional microwave-induced magnetoresistance oscillations in a dc-driven two-dimensional electron system

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Microwave-induced nonlinear magnetoresistance in a dc-driven two-dimensional electron system is examined using a multi-photon-assisted transport scheme directly controlled by the current. It is shown that near the second subharmonic of the cyclotron resonance, the frequency of the resistivity oscillation with the magnetic-field-normalized current density is double that at the cyclotron resonance and its harmonics, in excellent agreement with recent experimental findings by Hatke *et al.* [Phys. Rev. Lett. **101**, 246811 (2008)]. The current-induced alternative emergence of resonant two-photon and single-photon processes is responsible for this frequency doubling. Near the third subharmonic of the cyclotron resonance, the current-induced consecutive appearance of resonant 0-/3-photon, two-photon, and single-photon processes may lead to the frequency tripling of the resistivity oscillation.

I. INTRODUCTION

A number of extraordinary magnetotransport phenomena of irradiated two-dimensional (2D) electrons in very high Landau levels (LLs), especially the microwave-induced resistance oscillation (MIRO) and the zero-resistance state, have been the focus of intensive experimental[1–14] and theoretical[15–30] studies in the past few years.

Under the irradiation of a moderate microwave of frequency ω, the most striking resistance oscillations show up in the linear photoresistivity as a function of the inverse magnetic field $1/B$, featuring the periodical appearance of peak-valley pairs around $\omega/\omega_c = 1, 2, 3 \ldots$ ($\omega_c = eB/m$ is the cyclotron frequency and m is the electron effective mass). The cyclotron resonance and its harmonics are node points of peak-valley pairs, where the photoresistivity vanishes. In addition, secondary peak-valley pairs were also observed in the linear resistance near certain fractional values, $\omega/\omega_c = 1/2$, 1/3, 2/3, and 3/2. These fractional microwave-induced resistance oscillations were referred to real multiphoton participating processes.[31] They were also referred to higher harmonic effects,[5] the resonant series of consecutive single-photon transitions,[32,33] or microwave-induced sidebands.[34] Despite the fact that different scenarios have been proposed for these fractional MIROs of linear photoresistivity, they have so far been theoretically demonstrated to clearly appear in a real system only within the multiphoton-assisted scattering model.[31]

Another notable effect recently discovered is the significant role of a dc current. A relatively weak current alone, without microwave radiation, can also induce substantial magnetoresistance oscillations and zero-differential resistance states.[35–43] Simultaneous application of a finite dc current and microwave radiation leads to very interesting and complicated oscillatory behavior of resistance and differential resistance.[44–48] These facts clearly indicate that microwave-irradiated nonlinear magnetotransport is remarkably different from the linear one. So far experimental and theoretical investigations on nonlinear MIROs have been carried out in the magnetic field range lower than the cyclotron resonance and never anticipated any anomaly elsewhere. Very recent experiments by Hatke *et al.*[49] disclosed sharply distinct nonlinear transport behavior in the subharmonic fields of the cyclotron resonance. Particularly, they found that at the second cyclotron resonance subharmonic, $\omega/\omega_c = 1/2$, the differential resistivity oscillates as a function of the current density in the same way as that at the cyclotron resonance, $\omega/\omega_c = 1/2$, despite a factor of two difference in magnetic fields. In other words, at $\omega/\omega_c = 1/2$ the frequency of the resistivity oscillation with the magnetic-field-normalized current density is twice that at the cyclotron resonance and its harmonics. This is in vast contrast with the observed behavior of the resistivity oscillations in the magnetic field regimes of cyclotron resonance and its harmonics ($\omega/\omega_c \geq 1$). A nonlinear theoretical analysis capable of identifying the specific mechanism responsible for fractional MIROs and frequency doubling is highly desirable.

This paper presents such an analysis on microwave-induced nonlinear magnetotransport in high Landau levels in GaAs-based 2D heterosystems. With the microscopic balance-equation scheme directly controlled by the current, we are able to constitute a unified description for both the cyclotron resonance harmonic and subharmonic regimes of the magnetic fields and predict the resistivity oscillations with applied current density in excellent agreement with the experimental observation in both regimes.

II. FORMULATION

Our examination is based on a current-controlled scheme of photon-assisted transport,[19] which deals with a 2D system of short thermalization time having N_s electrons in a unit area of the x-y plane. These electrons, subjected to a uniform magnetic field $\boldsymbol{B} = (0, 0, B)$ in the z direction and scattered by random impurities and by phonons in the lattice, perform an integrative drift motion when an incident microwave electric field $\boldsymbol{E}_{is} \sin \omega t$ irradiates on and a current flows in the plane. In terms of the center-of-mass (c.m.) momentum and coordinate defined as $\boldsymbol{P} \equiv \Sigma_j \boldsymbol{p}_{j\parallel}$ and $\boldsymbol{R} \equiv N_s^{-1} \Sigma_j \boldsymbol{r}_j$, with $\boldsymbol{p}_{j\parallel} \equiv (p_{jx}, p_{jy})$ and $\boldsymbol{r}_j \equiv (x_j, y_j)$ being the momentum and coordi-

nate of the jth electron in the 2D plane and the relative electron momentum and coordinate $\boldsymbol{p}'_{j\parallel} \equiv \boldsymbol{p}_{j\parallel} - \boldsymbol{P}/N_s$ and $\boldsymbol{r}'_j \equiv \boldsymbol{r}_j - \boldsymbol{R}$, the Hamiltonian of this system can be written as the sum of a c.m. part $H_{\text{c.m.}}$ and a relative electron part H_{er},[50]

$$H_{\text{c.m.}} = \frac{1}{2N_s m}[\boldsymbol{P} - N_s e \boldsymbol{A}(\boldsymbol{R})]^2 - N_s e(\boldsymbol{E}_0 + \boldsymbol{E}_t) \cdot \boldsymbol{R}, \quad (1)$$

$$H_{\text{er}} = \sum_j \left\{ \frac{1}{2m}[\boldsymbol{p}'_{j\parallel} - e\boldsymbol{A}(\boldsymbol{r}'_j)]^2 + \frac{p_{jz}^2}{2m_z} + V(z_j) \right\} \\ + \sum_{i<j} V_c(\boldsymbol{r}'_i - \boldsymbol{r}'_j, z_i, z_j), \quad (2)$$

together with electron-impurity and electron-phonon couplings H_{ei} and H_{ep}. Here $\boldsymbol{A}(\boldsymbol{r}) = (-By, 0)$ is the in-plane component of the vector potential of the perpendicular magnetic field, $\boldsymbol{E}_0$ and $\boldsymbol{E}_t$ are the dc and ac components of the uniform electric field inside, m and m_z are, respectively, the electron effective mass parallel and perpendicular to the plane, $V(z)$ and $V_c(\boldsymbol{r}'_i - \boldsymbol{r}'_j, z_i, z_j)$ stand for the confined and Coulomb potentials.

The separation of the electron Hamiltonian into a c.m. part and a relative electron part amounts to look at electrons in the reference frame moving with their center of mass. The most important feature of this separation is that a spatially uniform electric field shows up only in $H_{\text{c.m.}}$ and that H_{er} is the Hamiltonian of a many-particle system subject to a magnetic field *without the electric field*. This enables us to deal with the relative electrons in the magnetic field *without tilting the Landau levels*.

We proceed with the Heisenberg operator equations for the rate of change in the c.m. velocity $\boldsymbol{V} = -i[\boldsymbol{R}, H]$: $\dot{\boldsymbol{V}} = -i[\boldsymbol{V}, H]$ and that of the relative electron energy H_{er}: $\dot{H}_{\text{er}} = -i[H_{\text{er}}, H]$. The c.m. coordinate $\boldsymbol{R}$ and velocity $\boldsymbol{V}$ in these equations can be treated classically, i.e., as the time-dependent expectation values of the c.m. coordinate and velocity, $\boldsymbol{R}(t)$ and $\boldsymbol{V}(t)$, such that $\boldsymbol{R}(t) - \boldsymbol{R}(t') = \int_{t'}^t \boldsymbol{V}(s)ds$. In the case of the steady transport under modest radiation of single frequency ω it suffices to assume that the c.m. velocity, i.e., the electron drift velocity, consists of a dc part $\boldsymbol{v}$ and a stationary time-dependent part of the form

$$\boldsymbol{V}(t) = \boldsymbol{v} + \boldsymbol{v}_1 \cos(\omega t) + \boldsymbol{v}_2 \sin(\omega t). \quad (3)$$

For high mobility and high carrier density 2D systems at low temperatures where effects of electron-impurity and electron-phonon scatterings are weak in comparison with internal thermalization of this many-electron system, it is a good approximation to carry out the statistical average of the above operator equations to leading orders in H_{ei} and H_{ep}. For this purpose we only need to know the distribution of relative electrons without being perturbed by H_{ei} or H_{ep}. The distribution function of the system described by Hamiltonian (2) without electric field is an isotropic Fermi-type with a single temperature T_e. Such a statistical average of the above operator equations yields the following force and energy balance equations in the steady state,

$$N_s e \boldsymbol{E}_0 + N_s e(\boldsymbol{v} \times \boldsymbol{B}) + \boldsymbol{F} = 0, \quad (4)$$

$$N_s e \boldsymbol{E}_0 \cdot \boldsymbol{v} + S_p - W = 0. \quad (5)$$

In this,

$$\boldsymbol{F} = \sum_{\boldsymbol{q}_\parallel} |U(\boldsymbol{q}_\parallel)|^2 \sum_{n=-\infty}^{\infty} \boldsymbol{q}_\parallel J_n^2(\xi) \Pi_2(\boldsymbol{q}_\parallel, \omega_0 - n\omega) \quad (6)$$

is the time-averaged damping force against c.m. motion, S_p is the time-averaged rate of the electron energy gain from the ac field, having an expression obtained from Eq. (6) by replacing the $\boldsymbol{q}_\parallel$ factor by $n\omega$. In Eq. (6), $U(\boldsymbol{q}_\parallel)$ is the effective impurity potential, $J_n(\xi)$ is the Bessel function of order n, $\Pi_2(\boldsymbol{q}_\parallel, \Omega)$ is the imaginary part of the electron-density correlation function at electron temperature T_e in the presence of the magnetic field, and $\omega_0 \equiv \boldsymbol{q}_\parallel \cdot \boldsymbol{v}$. The argument in $J_n(\xi)$ is $\xi \equiv \sqrt{(\boldsymbol{q}_\parallel \cdot \boldsymbol{v}_1)^2 + (\boldsymbol{q}_\parallel \cdot \boldsymbol{v}_2)^2}/\omega$. Note that although contributions of phonon scattering to $\boldsymbol{F}$ and S_p are neglected here, it provides the main channel for electron energy dissipation to the lattice with a time-averaged energy-loss rate W, having the expression as given in Ref. 19.

The ac components $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ of the electron drift velocity should be determined self-consistently from the incident ac field $\boldsymbol{E}_{\text{is}}$ by the Maxwell equations connecting both sides of the 2D system, taking into account the scattering-related damping forces $\boldsymbol{F}_s$ and $\boldsymbol{F}_c$.[19] However, for the high-mobility systems at low temperatures, the effects of these scattering-related forces are much weaker than that of radiative decay[30] and always negligible; hence $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are directly determined by the setup of the 2D system in the sample substrate.[19]

The effect of interparticle Coulomb interaction is included in the density correlation function to the degree of electron level broadening and screening (considered in the effective impurity and phonon potentials). The remaining $\Pi_2(\boldsymbol{q}_\parallel, \Omega)$ function in Eq. (6) is that of a noninteracting 2D electron gas in the magnetic field, which can be written in the Landau representation as[51]

$$\Pi_2(\boldsymbol{q}_\parallel, \Omega) = \frac{1}{2\pi l_B^2} \sum_{n,n'} C_{n,n'}(l_B^2 q_\parallel^2/2) \Pi_2(n, n', \Omega), \quad (7)$$

$$\Pi_2(n, n', \Omega) = -\frac{2}{\pi} \int d\varepsilon [f(\varepsilon) - f(\varepsilon + \Omega)] \\ \times \text{Im}\, G_n(\varepsilon + \Omega) \text{Im}\, G_{n'}(\varepsilon), \quad (8)$$

where $l_B = \sqrt{1/|eB|}$ is the magnetic length, $C_{n,n+l}(Y) \equiv n![(n+l)!]^{-1} Y^l e^{-Y}[L_n^l(Y)]^2$, with $L_n^l(Y)$ the associate Laguerre polynomial, $f(\varepsilon) = \{\exp[(\varepsilon - \mu)/T_e] + 1\}^{-1}$ is the Fermi function at electron temperature T_e, and $\text{Im}\, G_n(\varepsilon)$ is the density of states (DOS) of the broadened Landau level n.

The LL broadening results from impurity, phonon, and electron-electron scatterings. In the experimental ultraclean GaAs-based 2D systems having mobility higher than 10^3 m^2/V s the dominant elastic scatterings come mainly from residual impurities or defects in the background rather than from remote donors[52] and phonon and electron-electron scatterings are generally also not long ranged because of the screening. On the other hand, since the microwave-induced magnetoresistance oscillations occur at low temperatures and

low magnetic fields in high electron-density samples, the cyclotron radius of electrons involved in transport is generally much larger than the correlation length or the range of the dominant scattering potentials. In this case, the broadening of the LLs is expected to be a Gaussian form,[53] [$\varepsilon_n = (n + \frac{1}{2})\omega_c$ is the center of the nth LL],

$$\text{Im } G_n(\varepsilon) = -(2\pi)^{1/2}\Gamma^{-1} \exp[-2(\varepsilon - \varepsilon_n)^2/\Gamma^2] \quad (9)$$

with a $B^{1/2}$-dependent half width expressed as

$$\Gamma = (2\omega_c/\pi\tau_s)^{1/2}, \quad (10)$$

in which τ_s, the single-particle lifetime or quantum scattering time in the zero magnetic field, depends on impurity, phonon, and electron-electron scatterings.

Expressions (9) and (10) for the DOS of the nth level will be used in both the separated and overlapped LL regimes. The total DOS of a 2D system of unit area in the magnetic field, $g(\varepsilon) = -\Sigma_n \text{Im } G_n(\varepsilon)/\pi^2 l_B^2$, can be written as the sum of the DOS without magnetic field, g_0, and a magnetic-field-induced oscillatory part. When the LL widths 2Γ are much larger than the level spacing ω_c (heavily overlapped LLs), the oscillatory part is much smaller than g_0, and one can keep only its fundamental harmonic component that

$$g(\varepsilon) \approx g_0[1 - 2\delta\cos(2\pi\varepsilon/\omega_c)], \quad (11)$$

with $\delta = \exp(-\pi^2\Gamma^2/2\omega_c^2) = \exp(-\pi/\omega_c\tau_s)$.[54]

A reliable evaluation of single-particle lifetime τ_s (or the LL width Γ), which must include effects of impurity, phonon, and electron-electron scatterings, has not yet been available. We treat it as an empirical parameter or relate it to the zero-field linear mobility μ_0 or the transport relaxation time τ_{tr} using an empirical parameter α by $1/\tau_s = 4\alpha/\tau_{tr} = 4\alpha e/m\mu_0$.

III. NONLINEAR RESISTIVITY AND MULTIPHOTON PROCESSES

These formulations are convenient for current-driven nonlinear magnetotransport of any configuration. For an isotropic system where the frictional force $\boldsymbol{F}$ is in the opposite direction of the drift velocity $\boldsymbol{v}$ and the magnitudes of both the frictional force and the energy-loss rate depend only on $v \equiv |\boldsymbol{v}|$, we can write $\boldsymbol{F}(\boldsymbol{v}) = F(v)\boldsymbol{v}/v$ and $W(\boldsymbol{v}) = W(v)$. In the Hall configuration with velocity $\boldsymbol{v}$ in the x direction $\boldsymbol{v} = (v, 0, 0)$ or the current density $J_x = J = N_s ev$ and $J_y = 0$, Eq. (4) yields the transverse resistivity $R_{yx} = B/N_s e$ and the longitudinal resistivity R_{xx} and differential resistivity r_{xx} at given v as

$$R_{xx} = -F(v)/(N_s^2 e^2 v), \quad (12)$$

$$r_{xx} = -[\partial F(v)/\partial v]/(N_s^2 e^2). \quad (13)$$

In the linear case, the resistivity r_{xx} oscillations (peak-valley pairs) result from the resonant real photon-assisted electron transitions between different Landau levels.[31] We denote a real photon-assisted process in which an electron jumps across l LL spacings with the assistance (emission or absorption) of n photons as $n\omega : l\omega_c$ or $n:l$. This process con-

tributes, in the $r_{xx} - \omega/\omega_c$ curve, a pair structure consisting of a maximum and a minimum on both sides of $\omega/\omega_c = l/n$.

Thus, the single-photon process 1:1, the two-photon process 2:2, the three-photon process 3:3,… all contribute to the maximum-minimum pair around $\omega/\omega_c = 1$; the two-photon process 2:1, the four-photon process 4:2,… all contribute to the maximum-minimum pair around $\omega/\omega_c = 1/2$; the three-photon process 3:1, the six-photon process 6:2,… all contribute to the maximum-minimum pair around $\omega/\omega_c = 1/3$; etc. The virtual photon process (zero-photon process) during which the electron may absorb (emit) one or several photons and then emits (absorbs) them before finishing an intra-LL transition (assuming separated LLs for convenience), which can be denoted as 0:0, gives rise to an overall resistivity suppression without structure.[31]

The effects of a finite current show up as the energy (frequency) shift $\omega_0 = \boldsymbol{q}_\parallel \cdot \boldsymbol{v}$ in the density-correlation function $\Pi_2(\boldsymbol{q}_\parallel, \Omega)$ in Eq. (6). This energy shift indicates that an electron, having momentum $\boldsymbol{q}_\parallel$ and participating in the system integrative drift motion of velocity $\boldsymbol{v}$, carries an extra energy $\boldsymbol{q}_\parallel \cdot \boldsymbol{v}$ during its transition. In the case of low temperature (T_e much less than the Fermi energy ε_F) and large Landau-level filling factor ($\nu = \varepsilon_F/\omega_c \gg 1$), the major contributions to the summation in Eq. (7) come from terms $n \sim n' \sim \nu$, then the function $C_{n,n'}(x)$ has a sharp principal maximum near $x = 4\nu$. Therefore, as a function of the in-plane momentum $q_\parallel$, $\Pi_2(\boldsymbol{q}_\parallel, \Omega)$ sharply peaks around $q_\parallel \simeq 2k_F$. As the composite effect of this energy shift, the electron possesses an extra energy,

$$\omega_j \equiv 2k_F v = \sqrt{8\pi/N_s} J/e, \quad (14)$$

for its transition in addition to those provided by photons.

Therefore, the resonant condition applied in the linear case for electron scattering across l LL spacings with the assistance (emission or absorption) of n photons, $n\omega = l\omega_c$, should change to

$$\omega_j \pm n\omega = \pm l\omega_c \quad (n = 0, 1, 2, \ldots, \ l = 0, 1, 2, \ldots) \quad (15)$$

when the system has an integrative motion of velocity v, i.e., a finite current density $J = N_s ev$. We can still use symbol $n:l$ to denote this process. We will also use symbol n- to represent all the n-photon participating processes (electron may jump across any LL spacings).

Increase in the current density from zero reduces the rates of the scattering processes resonant in the linear case, such as those of zero-photon processes 0:0 everywhere, of single-photon process 1:1 and two-photon process 2:2 at $\omega/\omega_c = 1$, and of two-photon processes 2:1 and four-photon processes 4:2 at $\omega/\omega_c = 1/2$. But it may increase the rates of the scattering processes originally absent or very small in the linear case, such as those of single-photon process 1:1 and three-photon process 3:1 at $\omega/\omega_c = 1/2$.

At cyclotron resonance and its harmonics $\omega/\omega_c = 1, 2, 4, \ldots$, the resonant condition (15) can be satisfied only at

$$\epsilon_j \equiv \omega_j/\omega_c = 0, 1, 2, \ldots, \quad (16)$$

where the scattering rate peaks and the differential resistivity exhibits a maximum. Therefore, at these magnetic fields the
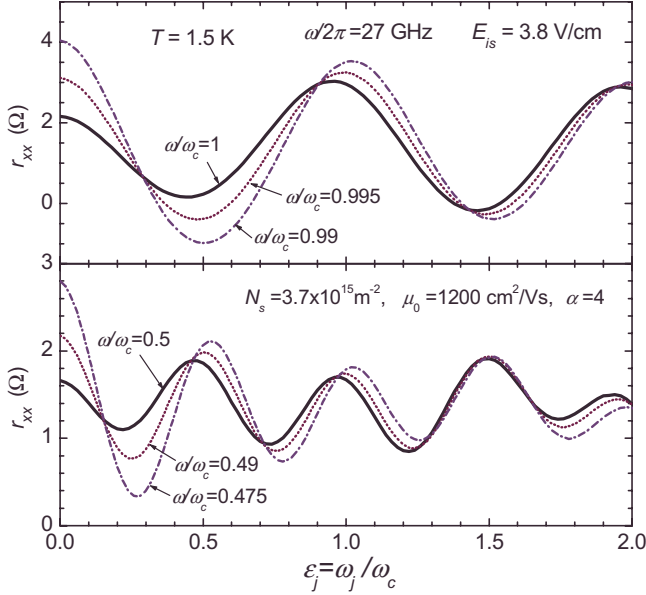
FIG. 1. (Color online) Magnetoresistivity r_{xx} vs $\epsilon_j = \omega_j/\omega_c$ at $\omega/\omega_c = 1$, 0.995, 0.99, 0.5, 0.49, and 0.475.

FIG. 2. (Color online) Magnetoresistivity r_{xx} vs ϵ_j at $\omega/\omega_c = 1/3$ and 0.5. Curves with 0–, 1–, 2–, 3–, and $m-$ are separated contributions from zero-photon, single-photon, two-photon, three-photon, and multiphoton processes.

r_{xx} oscillation with changing ϵ_j always has a periodicity $\Delta\epsilon_j \approx 1$.

The situation changes near subharmonic cyclotron resonant fields. For instance, at $\omega/\omega_c = 1/2$, the resonant condition (15) for zero-photon, two-photon, and four-photon processes ($n = 0$, 2, and 4) is satisfied at $\epsilon_j = 0, 1, 2, \ldots$, indicating that these even-photon processes contribute resistivity maxima there. At $\epsilon_j = 0.5, 1.5, \ldots$, on the other hand, the resonant condition (15) is satisfied for single-photon and three-photon processes ($n = 1$ and 3), indicating these odd-photon processes contribute resistivity maxima. Because of this, the period of r_{xx} oscillation with ϵ_j may shrink to $\Delta\epsilon_j \approx 0.5$ or the oscillation frequency doubles at $\omega/\omega_c = 1/2$.

At the third subharmonic field, $\omega/\omega_c = 1/3$, in addition to the appearance of resonant scattering of zero-photon and three-photon processes at $\epsilon_j = 0$, 1, and 2, the resonance scattering can also appear at $\epsilon_j = 1/3$, 4/3, and 5/3 for two-photon processes and at $\epsilon_j = 2/3$, 4/3, and 5/3 for single-photon processes, leading to possible shrinkage of the r_{xx} oscillation period to $\Delta\epsilon_j \approx 1/3$ or the frequency tripling.

These analyses are confirmed by the numerical results shown in Figs. 1 and 2, where we plot the differential resistivity r_{xx} calculated from Eq. (13) as a function of the normalized current density $\epsilon_j = \omega_j/\omega_c$ for a GaAs-based system of $N_s = 3.7 \times 10^{15}$ m^{-2} and $\mu_0 = 1200$ m^2/V s at $T = 1.5$ K, irradiated by a 27 GHz microwave having incident electric field amplitude $E_{is} = 3.8$ V/cm. The elastic scatterings are due to a mixture of short-range and background impurities

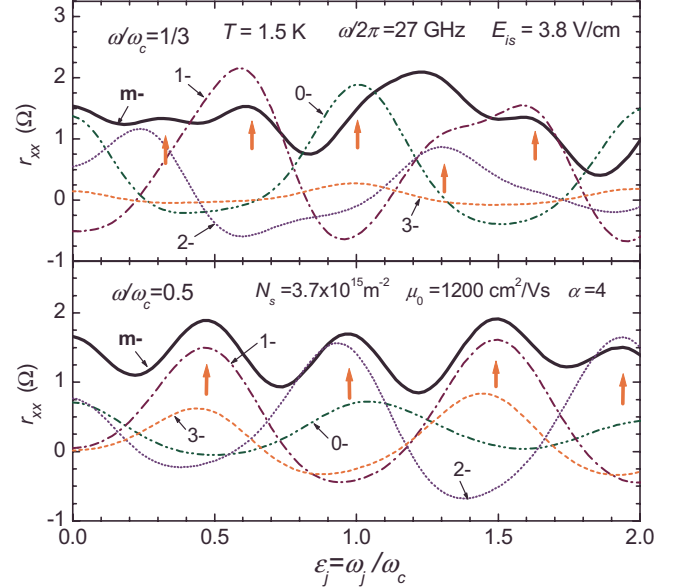and the broadening parameter $\alpha = 4$, or the LL width $2\Gamma/\omega_c \simeq 0.72(\omega/\omega_c)^{1/2}$, in the separated LL regime for $\omega/\omega_c < 1$. The predicted r_{xx} behaviors near $\omega/\omega_c = 1$ and $\omega/\omega_c = 0.5$ are in excellent agreement with recent experimental findings.[49] The r_{xx} oscillations at $\omega/\omega_c = 1/3$ exhibit the observable frequency tripling under this strength of microwave irradiation.

IV. SUMMARY

In summary, the electron in a dc-driven system having an integrative drift velocity v carries an extra average energy $\omega_j = 2k_F v$ in its transition between LLs, giving rise to a current-dependent resonance condition. The peaks of the differential resistivity r_{xx} arise from resonant multi-photon-assisted scatterings of electron jumping across l LL spacings. The current-induced consecutive emergence of different resonant multiphoton processes results in the frequency doubling and tripling of the resistance oscillations near the second and third subharmonic fields.

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